mt360_ur7_6_6_1 Faraday's Law

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Title

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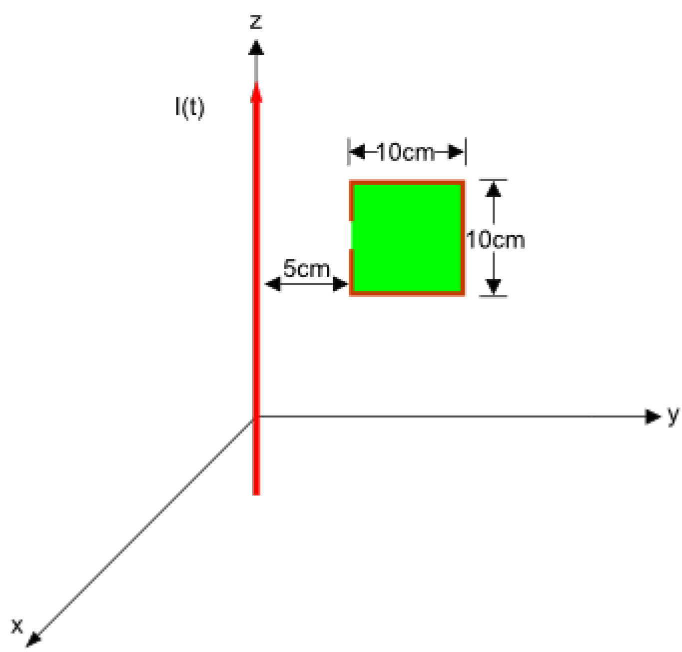
Description

A square loop with a small gap shown in the image below is coplanar with a long, straight wire carrying a time varying current. The time varying current is

$$I(t) = 5\cos 2\pi e^4 t (A)$$

For the purposes of the assignment, complete the exercises below by only analysing time from t = 0s to t = T with a time step of 1e-7s where T is the period of the current signal (I).



The gap drawn in the image was purposefully drawn large to increase its visibility. However, assume that the gap is negligibly small.

Exercise

In this exercise, do not use any types of loops.

1. Calculate the magnetic flux moving through the loop as a function of time. See the Useful Information section for help.
2. Determine the direction and magnitude of the current that would flow through a 4 Ohm resistor connected across the gap. The loop has an internal resistance of 1 Ohm.

Questions

1. What is the direction and magnitude of the current flowing through the 4 Ohm resistor from exercise 2.

Useful Information

Faraday's Law

The magnetic flux density **B** induced by a constant current is given by

$$B = \hat{\phi}\frac{\mu_0 I}{2\pi r} \qquad (Wb/m^2)$$

$p\hat{h}i$ is the direction of the magnetic fields penetrating an area, μ_0 is the permeability of the area the magnetic fields are travelling through, and r is the distance away from the current carrying wire.

Magnetic flux going through an area is the magnetic flux density integrated over the area.

$$\phi = \int_s B \cdot ds \qquad (Wb)$$

The electromagnetic force (emf), Vemf is related to a time varying flux by

$$V_{emf} = -N\frac{d\phi}{dt}$$

N is the number of conducting loops, and ϕ is the flux through the wire.

Approximating an integral

In this assignment, you need to calculate the magnetic flux through the area. In order to do this, you need to integrate the magnetic field density over the area. This calculation is simple after some algebraic manipulation.

$$\phi = \int_s B \cdot ds \qquad (Wb)$$

$$\phi = \int_s \hat{\phi} \frac{\mu_0 I}{2\pi r} \cdot ds \qquad (Wb)$$

The magnetic field is travelling through the loop in the negative x direction. This indicates that $\hat{\phi}$ is $-\hat{x}$.

$$\phi = \int_s -\hat{x} \frac{\mu_0 I}{2\pi r} \cdot ds \qquad (Wb)$$

The integral is only over the area. This allows the current to be pulled out of the integral.

$$\phi = I \int_s -\hat{x} \frac{\mu_0}{2\pi r} \cdot ds \qquad (Wb)$$

The area of interest is in the zy plane. Note that the radius ,r, in the above equation is a component of y. Thus the above equation can be changed to

$$\phi = I \int_{5cm}^{15cm} \int_z^{z+10cm} -\hat{x} \frac{\mu_0}{2\pi y} dz dy \qquad (Wb)$$

Since nothing inside the integral depends on z, we can pull everything outside of the integral over z.

$$\phi = I \int_{5cm}^{15cm} -\hat{x} \frac{\mu_0}{2\pi y} dy \int_z^{z+10cm} dz \qquad (Wb)$$

After evaluating the integral over z, the equation simplifies to

$$\phi = I \int_{5cm}^{15cm} -\hat{x} \frac{\mu_0}{2\pi y} 10e^{-2} dy \qquad (Wb)$$

An integral can be approximated by a summation when the steps, dy in this case, are small.

$$\phi = I \sum_{5cm}^{15cm} -\hat{x} \frac{\mu_0}{2\pi y} 10^{e^{-2}} dy \qquad (Wb)$$

Derivative

Taking the derivative of a function is measuring the slope of a line. In this assignment, you will take the derivative of the magnetic flux with respect to time.

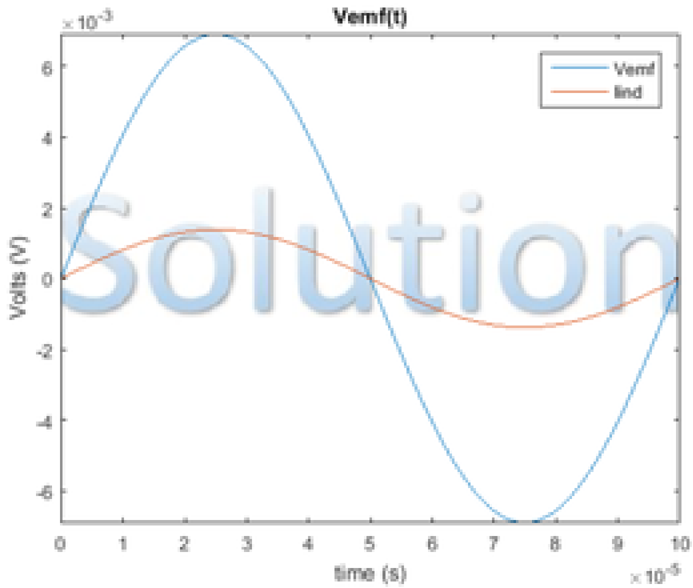
$$\frac{d\phi}{dt} = \frac{\phi(t+dt) - \phi(t)}{dt}$$

How to take a derivative without the use of a loop is up to you.

Provided Code

```
param

% Parameters
L.height = 10e-2;       % Height of the loop, m
L.width = 10e-2;        % Width of the loop, m
L.d = 5e-2;             % Distance of the loop's left side from the
 wire, m
L.R = 1;                % Internal resistance of wire, Ohms
R = 4;                  % Resistance of the resistor placed across the
 loop's gap.
f = 1e4;                % Frequency of the current, Hz
T = 1/f;                % Period of the current signal, s
A = 5;                  % Magnitude of the current signal, A
dt = 1e-7;              % Time step, s
t = 0:dt:T;             % Time array, s
I = A*cos(2*pi*f*t);    % Current through the wire, A
dy = 1e-4;              % Distance step, m
r = L.d:dy:L.width+L.d; % An array that represents the different radii
 inside the loop, m

% Part 1)
% Calculate the Flux through the area
Flux = % INSERT CODE HERE

% Calculate the Vemf across the small gap in the loop.
Vemf = % INSERT CODE HERE

% Part 2)
% Calculate the induced current through the 4 Ohm resistor
I_induced = % INSERT CODE HERE

% Plot
figure(1),clf;
plot(t(1:length(t)-1),Vemf,t(1:length(t)-1),I_induced);
legend('Vemf', 'Iind')
title('Vemf(t)')
xlabel('time (s)')
ylabel('Volts (V)')
```

Solution



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