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# mt360\_ur7\_4\_8\_1 Boundary Conditions

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## Title

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## Description

With reference to Fig. 4-19, pg 206 in the book.

$$\mathbf{E}_2 = \hat{x}3 - \hat{y}2 + \hat{z}(V/m), \epsilon_1 = 2\epsilon_0, \epsilon_2 = 18\epsilon_0$$

and the boundary has a surface charge density

$$\rho_s = 3.54 * 10^{-11} (C/m^2)$$

## Exercise

1. Complete the function "electricFieldBoundaryConditions" in order to calculate E1, the angle E2 makes with the z-plane, and the angle E1 makes with the z-plane. This function is at the bottom of the Provided Code section. You do not need to alter any other code.
2. Play around with the interaction to answer the questions.

## Questions

1. From part 1, what is E1, and what angle does E2 make with the z-plane?
2. From part 2, what affect does changing epsilon1, epsilon2, and rho\_s have on E1? Why?

## Useful Information

**tangential components**

The tangential component runs parallel to the boundary.  
In this exercise, the x and y components of E1 and E2  
compose the tangential component.

$$E_t = \sqrt{E_x^2 + E_y^2}$$

#### Angle

The angle the electric field makes with the z-plane is  
determined by the tangential and normal component of E.  
In this exercise, the normal component is z.

$$\tan \theta = \frac{E_t}{E_n}$$

#### Boundary Conditions for the electric fields

| <i>FieldComponent</i> | <i>AnyTwoMedia</i>                                      |
|-----------------------|---------------------------------------------------------|
| <i>Tangential E</i>   | $E_{1t} = E_{2t}$                                       |
| <i>Tangential D</i>   | $\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$ |
| <i>Normal E</i>       | $\epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$        |
| <i>Normal D</i>       | $D_{1n} - D_{2n} = \rho_s$                              |

## Provided Code

```
function boundary_conditions()  
param  
  
% Parameters  
  
var.P.Eo = P.Eo; % Electric permittivity of free space  
var.E2.x = 3; % x component, m  
var.E2.y = -2; % y component, m  
var.E2.z = 2; % z component, m  
var.E2.epsilon = 18*P.Eo; % Electric permittivity, F/m  
var.E2.Et = 0; % Tangential component, m  
var.E2.theta = 0; % Angle vector E2 makes with the z-plane,  
% degrees  
  
% These parameters are zero because they have not been computed yet.  
var.E1.x = 0; % x component, m  
var.E1.y = 0; % y component, m  
var.E1.z = 0; % z component, m  
var.E1.epsilon = 2*P.Eo; % Electric permittivity, F/m  
var.E1.Et = 0; % Tangential component, m  
var.E1.theta = 0; % Angle vector E1 makes with the z  
% plane, degrees  
  
var.rho_s = 3.54e-11; % Surface charge density at  
% boundary, C/m^2
```

```
% Call the function at the beginning in order to initialize data
[var.E1,var.E2] = ...
electricFieldBoundaryConditions(var.E2,var.E1,var.rho_s);

epsilon1 = 2;
epsilon2 = 18;
rho = 3.45;

% The code below is used to create the interactive gui

f = figure(1),clf();
var.ax = axes('Parent',f,'position',[0.1 0.5 0.8 0.4]);
var.ax.YLim = [-4,22]);

% plot initial data
h = plot([0,var.E1.Et],[0,var.E1.z],[-[0,var.E2.Et],[-[0,var.E2.z]]);
legend('E1','E2','location','NorthWest')
title('Boundary Conditions')
var.txt1 = text(1,0,['E1 theta = ', num2str(var.E1.theta),'
    degrees']);
var.txt2 = text(1,-2,['E2 theta = ', num2str(var.E2.theta),'
    degrees']);

% User interactive controls
e1_ctrl = uicontrol('Parent',f,'Style','slider',...
    'Position',[60,20,420,20],'value',epsilon1,...
    'min',2,'max',20,'Callback',{@update_e1,h},...
    'Tag','slider1');
e1_ctrl.UserData = var;
e2_ctrl = uicontrol('Parent',f,'Style','slider',...
    'Position',[60,40,420,20],'value',epsilon2,'min',2,...
    'max',20,'Callback',{@update_e2,h});
rho_ctrl = uicontrol('Parent',f,'Style','slider',...
    'Position',[60,60,420,20],'value',rho,...
    'min',0,'max',20,'Callback',{@update_rho_s,h});

% controls to display slider names
e1_txt = uicontrol('Parent',f,'Style','text',...
    'Position',[0,20,60,20],'String','epsilon1');
e2_txt = uicontrol('Parent',f,'Style','text',...
    'Position',[0,40,60,20],'String','epsilon2');

    rho_txt = uicontrol('Parent',f,'Style','text',...
        'Position',[0,60,60,20],'String','rho');

% controls to display slider values
e1_disp = uicontrol('Parent',f,'Style','text',...
    'Position',[480,16,20,15],'String',...
    num2str(e1_ctrl.Value),'Tag','e1_disp');
e2_disp = uicontrol('Parent',f,'Style','text',...
    'Position',[480,37,20,15],'String',...
    num2str(e2_ctrl.Value),'Tag','e2_disp');
rho_disp = uicontrol('Parent',f,'Style','text',...
    'Position',[480,59,80,15],'String',...
    [num2str(rho_ctrl.Value),' x10^-11'],'Tag','rho_disp');
```

```
var.ax.YLim = ([-4,22]);
end

% Callback function when epsilon1 slider changes
function update_e1(hObject,callbackdata,h)
    var = hObject.UserData;
    var.E1.epsilon = (hObject.Value)*var.P.Eo;
    [var.E1,var.E2] = electricFieldBoundaryConditions(...
        var.E2,var.E1,var.rho_s);
    set(h(1,1), 'YData',[0,var.E1.z]);
    set(var.txt1, 'String',['E1 theta = ',...
        num2str(var.E1.theta),' degrees'])
    set(var.txt2, 'String',['E2 theta = ',...
        num2str(var.E2.theta),' degrees'])
    hObject.UserData = var;
    var.ax.YLim = ([-4,22]);
    e1_disp = findobj('Tag','e1_disp');
    set(e1_disp,'String',num2str(hObject.Value));
end

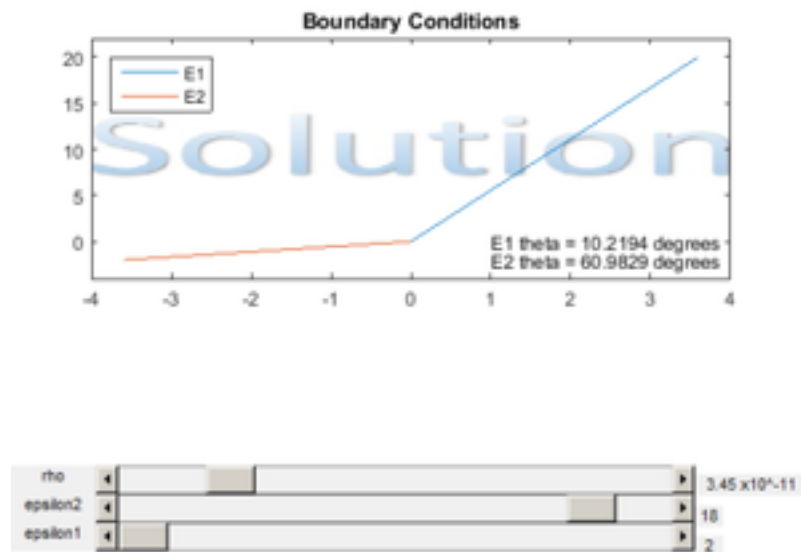
% Callback function when epsilon2 slider changes
function update_e2(hObject,callbackdata,h)
    e1_h = findobj('Tag','slider1');
    var = e1_h.UserData;
    var.E2.epsilon = (hObject.Value)*var.P.Eo;
    [var.E1,var.E2] = electricFieldBoundaryConditions(...
        var.E2,var.E1,var.rho_s);
    set(h(1,1), 'YData',[0,var.E1.z]);
    set(var.txt1, 'String',['E1 theta = ',...
        num2str(var.E1.theta),' degrees'])
    set(var.txt2, 'String',['E2 theta = ',...
        num2str(var.E2.theta),' degrees'])
    guidata(hObject, var);
    var.ax.YLim = ([-4,22]);
    e2_disp = findobj('Tag','e2_disp');
    set(e2_disp,'String',num2str(hObject.Value));
end

% Callback function when rho slider changes
function update_rho_s(hObject,callbackdata,h)
    e1_h = findobj('Tag','slider1');
    var = e1_h.UserData;
    var.rho_s = (hObject.Value)*1e-11;
    [var.E1,var.E2] = electricFieldBoundaryConditions(...
        var.E2,var.E1,var.rho_s);
    set(h(1,1), 'YData',[0,var.E1.z]);
    set(var.txt1, 'String',['E1 theta = ',...
        num2str(var.E1.theta),' degrees'])
    set(var.txt2, 'String',['E2 theta = ',...
        num2str(var.E2.theta),' degrees'])
    guidata(hObject, var);
    var.ax.YLim = ([-4,22]);
    rho_disp = findobj('Tag','rho_disp');
    set(rho_disp,'String',[num2str(hObject.Value),' x10^-11']);
```

end

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%           Students only need to alter the function below  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
  
% This function needs to be completed by the user.  
function [E1,E2] = electricFieldBoundaryConditions(E2,E1,rho_s)  
  
E2.Et = % INSERT CODE HERE           % Tangential component of E2, m  
E2.theta = % INSERT CODE HERE        % The angle E2 makes  
                                         % with the z-axis, degrees  
  
E1.x = % INSERT CODE HERE            % X-component of E1, m  
E1.y = % INSERT CODE HERE            % Y-component of E1, m  
E1.Et = E2.Et;                       % Tangential component of E1, m  
E1.z = % INSERT CODE HERE            % The angle E1 makes  
                                         % with the z-axis, degrees  
  
E1.theta = atan(E1.Et/E1.z)*180/pi;  % The angle E1 makes  
                                         % with the z-axis, degrees  
  
end
```

## Solution



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