
MT240_NR_8_2_1 RLC Circuit

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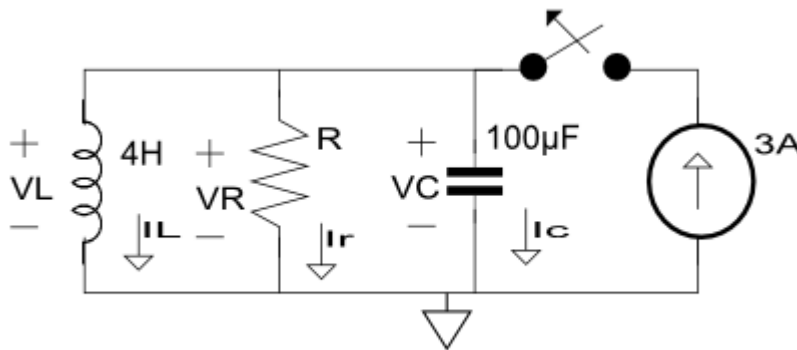
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Title

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Description

You have a parallel RLC circuit as shown in the image below. The current source has a value of 3A, the capacitor 100 μ F, the inductor 4H, and a potentiometer (R) can assume values from 70 Ohms to 1070 Ohms with increments of 200 Ohms. Assume that the switch has been closed for a long time before opening it at $t = 0$ s.



Exercise

1. Calculate the current through the inductor as a function of time from $t = 0$ s to $t = 1$ s with time steps of 10ms for every resistor value. See the section **Useful Information** for help.
2. Plot the current through the inductor as a function of time for all values of R.

Questions

1. How does resistance affect the current? Why? Hint, think back to the MATLAB assignment `mt240_nr_7_2_1_nat_tesp_rc_circuit`.

Useful Information

For the purposes of this assignment we will work with the natural response of an over-damped circuit modeled by

$$A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$s^2 + \alpha s + \omega_o^2 \text{ Characteristic Equation}$$

$$s_1, s_2 = \text{roots of the Characteristic Equation}$$

$$x(0) = A_1 + A_2$$

$$\frac{dx}{dt}(0) = A_1 s_1 + A_2 s_2$$

Roots

MATLAB has a built in command called `roots`. It takes in a polynomial and returns the roots of the polynomial. For example consider the polynomial

$$x^2 + 6x + 9 = 0$$

To find the roots of the polynomial I would use the `roots` command.

```
r = roots([1 6 9])
```

Note that the most right digit has an order of 0, the middle digit has an order of 1 and the left most digit has an order of 2. The command computes the roots of the polynomial and stores them in `r`.

Finding A1 and A2

After using the roots command to find s_1 and s_2 , we can set up a matrix to solve for A_1 and A_2 . Don't you just love matrices by now!

We begin by using the two systems of equations stated earlier.

$$x(0) = A_1 + A_2$$

$$\frac{dx}{dt}(0) = A_1 s_1 + A_2 s_2$$

We can put them systems of equations into matrix form.

Matrix S Matrix A Matrix D

$$\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} * \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} x(0) \\ \frac{dx}{dt}(0) \end{bmatrix}$$

Solve for matrix B

$$\begin{aligned} SA &= D \\ S^{-1}SA &= S^{-1}D \\ A &= S^{-1}D \\ A_1 &= A(1) \\ A_2 &= A(2) \end{aligned}$$

Provided Code

```
% Parameters
C = 100e-6;           % Value of the capacitor, F
L = 4;                % Value of the inductor, H
R = 70:200:1070;      % Resistor values, Ohms
t = 0:.01:1;          % Time array, s

%Initial conditions at t = 0+
VcInit = % INSERT CODE HERE; % Voltage across the capacitor, V
IcInit = % INSERT CODE HERE; % Current through the capacitor, A
VrInit = % INSERT CODE HERE; % Voltage across the resistor, V
IrInit = % INSERT CODE HERE; % Current through the resistor, A
VlInit = % INSERT CODE HERE; % Voltage across the inductor, V
IlInit = % INSERT CODE HERE; % Current through the inductor, A

%Final conditions at t = inf+
VcFinal = % INSERT CODE HERE; % Voltage across the capacitor
IcFinal = % INSERT CODE HERE; % Current through the capacitor
```

```
VrFinal = % INSERT CODE HERE;    % Voltage across the resistor
IrFinal = % INSERT CODE HERE;    % Current through the resistor
VlFinal = % INSERT CODE HERE;    % Voltage across the inductor
IlFinal = % INSERT CODE HERE;    % Current through the inductor

% Calculate the derivative
dIdt = VlInit/L;                % The derivative of the
                                % voltage across the capacitor
D = [IlInit;dIdt];              % D matrix

% Calculate coefficients of the characteristic equation
% polynomial
%  $d^2i/dt + \alpha di/dt + \omega^2 = 0$ 
%  $\Rightarrow s^2 + \alpha s + \omega^2 = 0$ 
%  $\Rightarrow a*s^2 + b*s + c = 0$ 
alpha = 1./(2*R*C);
omega = 1/sqrt(L*C);
a = 1;                          %  $s^2$  coefficient, has order 2
b = alpha;                       %  $s$  coefficient, has order 1
c = omega^2;                     % last coefficient, has order 0

figure(1);
for m = 1:length(R)              % The for loop goes through all
                                % values of the resistor
r = % INSERT CODE HERE;          % Calculate the roots

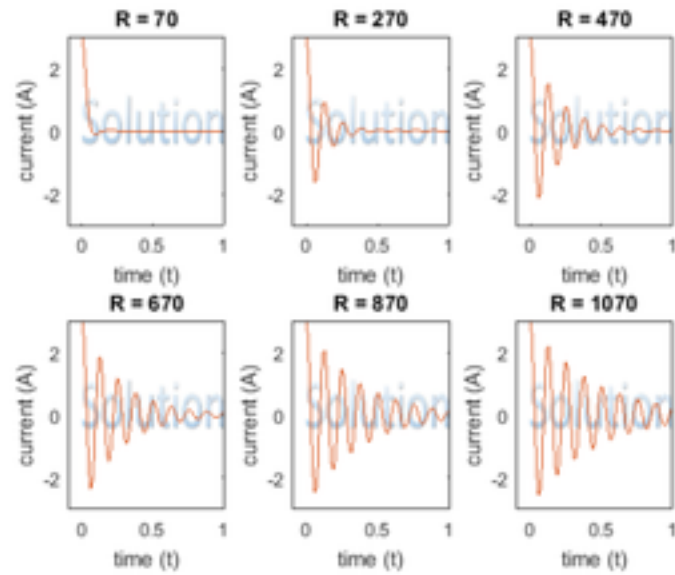
S = % INSERT CODE HERE;          % Form the S matrix
Sinv = inv(S);                   % Calculate the inverse of the S
                                % matrix
A = Sinv*D;                      % Calculate the A matrix
A1 = A(1);                       % Grab the coefficients
A2 = A(2);

% Calculate the current through the inductor as
% a function of time.
I = % INSERT CODE HERE;

% Plot the current
subplot(2, 3, m);

plot(t,I);
title(['R = ', int2str(R(m))]);
xlabel('time (t)');
ylabel('current (A)');
axis([-1 1 -3 3]);
end
```

solution



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