

MT240_NR_8_2_1 RLC Circuit

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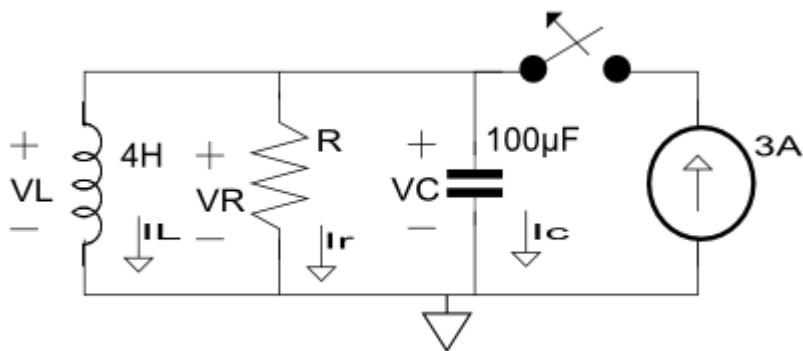
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Title

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Description

You have a parallel RLC circuit as shown in the image below. The current source has a value of 3A, the capacitor $100\mu\text{F}$, the inductor 4H , and a potentiometer (R) can assume values from 70 Ohms to 1070 Ohms with increments of 200 Ohms. Assume that the switch has been closed for a long time before opening it at $t = 0\text{s}$.



Exercise

1. Calculate the current through the inductor as a function of time from $t = 0\text{s}$ to $t = 1\text{s}$ with time steps of 10ms for every resistor value. See the section **Useful Information** for help.
2. Plot the current through the inductor as a function of time for all values of R .

Questions

1. How does resistance affect the current? Why? Hint, think back to the MATLAB assignment mt240_nr_7_2_1_nat_tesp_rc_circuit.

Useful Information

For the purposes of this assignment we will work with the natural response of an over-damped circuit modeled by

$$A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\alpha = \frac{1}{2RC}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$s^2 + \alpha s + \omega_o^2 \text{ Characteristic Equation}$$

s_1, s_2 = roots of the Characteristic Equation

$$x(0) = A_1 + A_2$$

$$\frac{dx}{dt(0)} = A_1 s_1 + A_2 s_2$$

Roots

MATLAB has a built in command called roots. It takes in a polynomial and returns the roots of the polynomial. For example consider the polynomial

$$x^2 + 6x + 9 = 0$$

To find the roots of the polynomial I would use the roots command.

```
r = roots([1 6 9])
```

Note that the most right digit has an order of 0, the middle digit has an order of 1 and the left most digit has an order of 2. The command computes the roots of the polynomial and stores them in r.

Finding A1 and A2

After using the roots command to find s_1 and s_2 , we can set up a matrix to solve for A_1 and A_2 . Don't you just love matrices by now!

We begin by using the two systems of equations stated earlier.

$$x(0) = A_1 + A_2$$

$$\frac{dx}{dt(0)} = A_1 s_1 + A_2 s_2$$

We can put them systems of equations into matrix form.

Matrix S Matrix A Matrix D

$$\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} * \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} x(0) \\ \frac{dx}{dt(0)} \end{bmatrix}$$

Solve for matrix B

$$\begin{aligned} SA &= D \\ S^{-1}SA &= S^{-1}D \\ A &= S^{-1}D \\ A_1 &= A(1) \\ A_2 &= A(2) \end{aligned}$$

Provided Code

```
% Parameters
C = 100e-6;           % Value of the capacitor, F
L = 4;                 % Value of the inductor, H
R = 70:200:1070;       % Resistor values, Ohms
t = 0:.01:1;           % Time array, s

%Initial conditions at t = 0+
VcInit = % INSERT CODE HERE;      % Voltage across the capacitor, V
IcInit = % INSERT CODE HERE;      % Current through the capacitor, A
VrInit = % INSERT CODE HERE;      % Voltage across the resistor, V
IrInit = % INSERT CODE HERE;      % Current through the resistor, A
ViInit = % INSERT CODE HERE;      % Voltage across the inductor, V
IlInit = % INSERT CODE HERE;      % Current through the inductor, A

%Final conditions at t = inf+
VcFinal = % INSERT CODE HERE;     % Voltage across the capacitor
IcFinal = % INSERT CODE HERE;     % Current through the capacitor
```

```
VrFinal = % INSERT CODE HERE; % Voltage across the resistor
IrFinal = % INSERT CODE HERE; % Current through the resistor
VlFinal = % INSERT CODE HERE; % Voltage across the inductor
IlFinal = % INSERT CODE HERE; % Current through the inductor

% Calculate the derivative
dIdT = VlInit/L; % The derivative of the
% voltage across the capacitor
D = [IlInit;dIdT]; % D matrix

% Calculate coefficients of the characteristic equation
% polynomial
%  $d^2i/dt^2 + \alpha di/dt + \omega^2 = 0$ 
%  $\Rightarrow s^2 + \alpha s + \omega^2 = 0$ 
%  $\Rightarrow a s^2 + b s + c = 0$ 
alpha = 1./(2*R*C);
omega = 1/sqrt(L*C);
a = 1; % s^2 coefficient, has order 2
b = alpha; % s coefficient, has order 1
c = omega^2; % last coefficient, has order 0

figure(1);
for m = 1:length(R) % The for loop goes through all
% values of the resistor
r = % INSERT CODE HERE; % Calculate the roots

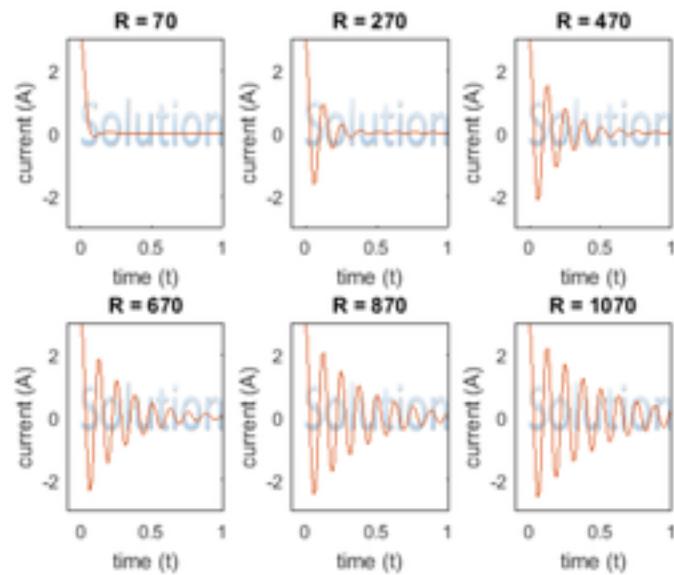
S = % INSERT CODE HERE; % Form the S matrix
Sinv = inv(S); % Calculate the inverse of the S
% matrix
A = Sinv*D; % Calculate the A matrix
A1 = A(1); % Grab the coefficients
A2 = A(2);

% Calculate the current through the inductor as
% a function of time.
I = % INSERT CODE HERE;

% Plot the current
subplot(2, 3, m);

plot(t,I);
title(['R = ', int2str(R(m))]);
xlabel('time (t)');
ylabel('current (A)');
axis([-0.1 1 -3 3]);
end
```

solution



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