
MT240_NR_6_3_1 Capacitor

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Title

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Description

A 5F capacitor with an initial voltage of 5V is attached to a current source. The behavior of the current source is given to you below.

```
% creates an array of length 56 that represents the
% current source as a function of time.
I = zeros(1,71); % Current array, A
t = 0:length(I)-1; % Time array,s
for m = 1:length(I)
    if m < 10
        I(m) = 0;
    elseif m < 20
        I(m) = -5 +.5*m;
    elseif m < 25
        I(m) = -15+m;
    elseif m < 35
        I(m) = 10;
    elseif m < 40
        I(m) = 115-3*m;
    elseif m < 45
        I(m) = 75-2*m;
    elseif m < 55
        I(m) = -15;
    elseif m < 70
        I(m) = -70+m;
    else
        I(m) = 0;
    end
end
```

Exercise

1. Calculate the voltage across the capacitor as a function of time by using the sum command to approximate the integral. See the section "Useful Information" for aid.
2. Calculate the power stored in the capacitor as a function of time.
3. Calculate the energy stored in the capacitor as a function of time.
4. Plot the capacitor's current, voltage, power, and energy as a function of time. Include titles and labels.

Questions

1. Approximate at what point in time the energy is zero in the capacitor? At this time, what is the capacitor doing? (discharging/charging).

Useful Information

Approximating Integration

Integration is used with continuous functions. Since computers cannot handle continuous functions that expand from $-\text{Inf}$ to $+\text{Inf}$, computers can only approximate integration over a finite period of time. The derivation of this approximation is shown below using the relationship between a capacitor's voltage and current.

$$V_c = \frac{1}{C} \int_{-\infty}^{\infty} i_c dt + V_i$$

- C is the capacitor's value. In this case it is 5F.
- Vi is the initial voltage across the capacitor at $t = 0$.

Since the current source only has non-zero values from $t = 0$ to $t = 70$, the bounds of integration can be changed from 0 to 70.

$$V_c = \frac{1}{C} \int_{0^+}^{70} i_c dt + V_i$$

Since computers can only handle discrete signals and not continuous signals. The integration becomes a summation, and dt becomes the time step between every discrete sample in the discrete signal array.

$$V_c = \frac{1}{C} \sum_{0^+}^{70} i_c \Delta t + V_i$$

For this specific assignment, since the time step is 1, the equation can be simplified.

$$V_c = \frac{1}{C} \sum_{0^+}^{70} i_c + V_i$$

The equation above calculates the voltage across the capacitor at $t = 70\text{s}$. However, part 1 of the assignment asks you to calculate the voltage across the capacitor as a function of time. To calculate the voltage at a specific point in time, you only need to change the bounds of summation as shown below assuming that the lower bound is $t = 0\text{s}$.

$$V_c(t = T) = \frac{1}{C} \sum_{0^+}^T i_c + V_i$$

The rest of part 1 is up to you to figure out. Hint, use a for-loop.

Power

Power is the product of the voltage and current.

$$P = VI$$

Energy

The energy stored in a capacitor has the following relationship.

$$E = \frac{1}{2} CV^2$$

Provided Code

```
% Generates the current array as a function of time.
I = zeros(1,71); % Allocation of the current array, A
t = 0:length(I)-1; % Time array, s
for m = 1:length(I)
    if m < 10
        I(m) = 0;
    elseif m < 20
        I(m) = -5 + .5*m;
    elseif m < 25
        I(m) = -15+m;
    elseif m < 35
        I(m) = 10;
    elseif m < 40
        I(m) = 115-3*m;
    elseif m < 45
        I(m) = 75-2*m;
    elseif m < 55
        I(m) = -15;
    elseif m < 70
        I(m) = -70+m;
```

```
else
    I(m) = 0;
end
end

% Parameters
Vi = 5; % Initial voltage across the capacitor, V
C = 5; % Value of the capacitor, F
V = zeros(1,71); % Allocates the voltage array, V

% Part 1)
% Calculate voltage as a function of time.
%INSERT CODE HERE

% Part 2)
% Calculate power as a function of time.
Power = %INSERT CODE HERE

% Part 3)
% Calculate energy as a function of time.
Energy = %INSERT CODE HERE

% Part 4)
% Plot the current, voltage, power, and energy as a function
% of time.
% %INSERT CODE HERE
```

Solution

