# MT240_NR_4_12_1 Max Power Transfer

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# Title

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# Description

The Norton or Thevenin equivalent circuit allows the designer to reduce a complex circuit into a simple model that predicts the behavior of the circuit from the perspective of a certain terminal. Often this is done when the designer wishes to know what load impedance to add onto the circuit to maximize power transfer to the load.

Power is the product of current and voltage. To maximize power, the designer needs to maximize voltage and current. However, when adding a load onto a circuit, there is a trade off between voltage and current. The higher the current through the load, the less voltage across the load and vice-versa. The dificulty is balancing the voltage and the current in order to maximize power.

A designer needs to know what impedance value allows for maximum power transfer by allowing the right amount of voltage across it and current through it.

# Exercise:

1. You are given the circuit as shown in **image 1** below. Reduce the circuit to its Thevenin equivalent circuit (See **image 2**). To do so, solve for the voltage open circuit (Voc) and the short circuit current (Isc). See the Useful Information and Provided Code sections to help you get started.

2. Using the calculated Voc and Isc, calculate the Thevenin resistance.

3. Hook the simplified circuit up to a potentiometer that can assume the values from 0 to 100 Ohms with increments of 1 Ohms.

4. Calculate the voltage, current and power across the potentiometer as a function of resistance.

5. Plot the calculated voltage, current, and power as a function of resistance. Include labels, a legend, and a title on the plot.

**Image 1**

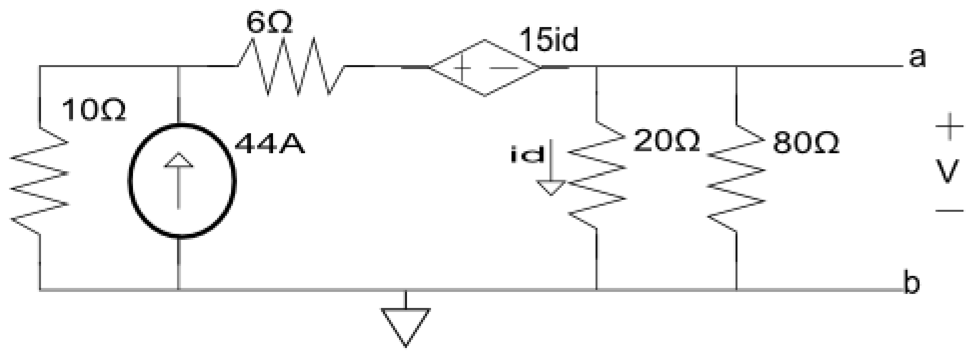
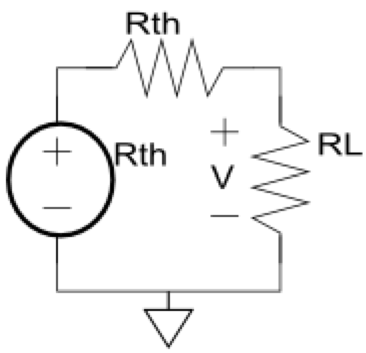



**Image 2**




# Questions

1. At what potentiometer value is the power transfer maximum, and how does it's value relate to Thevenin Impedance of the circuit.

# Useful Information

Since the main purpose of this assignment is to help you become familiar with MATLAB, you will be walked through the solution to finding the Thevenin Equivalent circuit.

The circuit can be solved using various techniques. This assignment will walk you through solving it using the node voltage method.

By inspection of the circuit, you can identify two unknown essential nodes: one above the current source, and the other above the 20 and 80 Ohm resistors. The node above the current source will be referred to as Vx, and the node above the 20 Ohm and 80 Ohm resistors will be referred to Voc since this is the open circuit voltage across the terminals a,b. For convenience, the resistors and current sources will be labelled as follows:

```
R1 = 10 Ohms
R2 = 6 Ohms
R3 = 20 Ohms
```

R4 = 80 Ohms
I1 = 44 Amps

iR2 is the current flowing through the resistor R2.

**Solve for Voc**

Supernode

$$\frac{Vx}{R1} - I1 + \frac{Voc}{R3} + \frac{Voc}{R4} = 0$$

$$\rightarrow Vx\left(\frac{1}{R1}\right) + Voc\left(\frac{1}{R3} + \frac{1}{R4}\right) = I1$$

Constraint

$$Vx - id(15) - Voc - iR2(R2) = 0$$

$$\rightarrow Vx + Voc(-1) + id(-15) + iR2(-R2) = 0$$

Dependent Source

$$id = \frac{Voc}{R3}$$

$$\rightarrow Voc\left(\frac{-1}{R3}\right) + id + iR2 = 0$$

Current Through the R2 Resistor

$$iR2 = \frac{Voc}{R3} + \frac{Voc}{R4}$$

$$\rightarrow Voc\left(\frac{-1}{R3} + \frac{-1}{R4}\right) + iR2 = 0$$

System of Equations

$$\begin{array}{ccccccccc}
Vx\left(\frac{1}{R1}\right) & + & Voc\left(\frac{1}{R3} + \frac{1}{R4}\right) & + & id(0) & + & iR2(0) & = & I1 \\
Vx(1) & + & Voc(-1) & + & id(-15) & + & iR2(-R2) & = & 0 \\
Vx(0) & + & Voc\left(\frac{-1}{R3}\right) & + & id(1) & + & iR2(1) & = & 0 \\
Vx(0) & + & Voc\left(\frac{-1}{R3} + \frac{-1}{R4}\right) & + & id(0) & + & iR2(1) & = & 0
\end{array}$$

Matrix Form

$$\begin{bmatrix} \frac{1}{R1} & \frac{1}{R3}+\frac{1}{R4} & 0 & 0 \\ 1 & -1 & -15 & -R2 \\ 0 & \frac{-1}{R3} & 1 & 1 \\ 0 & \frac{-1}{R3}+\frac{-1}{R4} & 0 & 1 \end{bmatrix} * \begin{bmatrix} Vx \\ Voc \\ id \\ iR2 \end{bmatrix} = \begin{bmatrix} I1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Let the first matrix be represented by R, the second by V, and the third by A. Then the above matrix equation can be represented in the form

$$R * V = A$$

Since the objective is to find Voc, you need to solve for matrix V

$$\begin{aligned} R * V &= A \\ R^{-1}RV &= R^{-1}A \\ V &= R^{-1}A \end{aligned}$$

Voc is the second element in the array V

$$Voc = V(2)$$

**Solve for Isc**

Solving for Isc is simple! Because of the short across terminals a and b, no current will flow through resistor R4 and R3. Since there is no current flowing through resistor R3, the dependent voltage source goes to zero. Thus resistors R3 and R4 can be removed from the circuit and the dependent voltage source can be replaced by a short. This greatly simplifies the produce to find Isc.

$$\frac{Vx}{R1} - I1 + \frac{Vx}{R2} = 0$$

$$Vx\left(\frac{1}{R1} + \frac{1}{R2}\right) = I1$$

$$Vx = I1\left(\frac{1}{R1} + \frac{1}{R2}\right)$$

$$Isc = \frac{Vx}{R2}$$

Note that the value of Vx in solving for Isc is not the same value of Vx in solving for Voc.

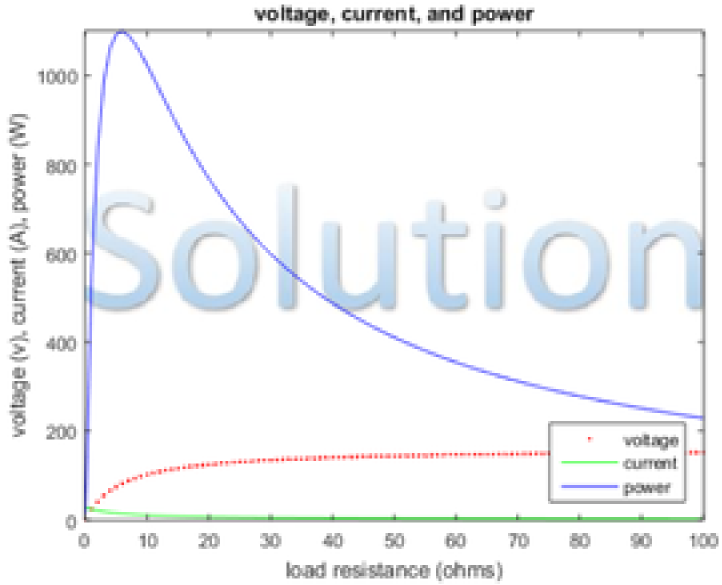
**Solve for Rth**

$$Rth = \frac{Voc}{Isc}$$

# Provided Code

```
% Parameters
R1 = 10; % Ohms
R2 = 6;  % Ohms
R3 = 20; % Ohms
R4 = 80; % Ohms
I1 = 44; % Amps
RL = 0:1:100; % Potentiometer, Ohms

% Part 1)
% Solve for Voc

% R Matrix
R = % INSERT CODE HERE

% A Matrix
A = [I1; 0; 0; 0];

% Solve for V Matrix
V = % INSERT CODE HERE;

% Extract Voc from V matrix
Voc = % INSERT CODE HERE;

% Assign Voc to Vth
Vth = Voc;

% Solve for ISC
Vx = (I1/(1/R1+1/R2));

% Isc = Vx/R2;
Isc = % INSERT CODE HERE;

% Part 2)
% solve for Rth
Rth = Vth/Isc;

% Part 3)
% Obtain power, voltage, and current across the potentiometer as a
 function
% of the potentiometer's impedances.
VL  = % INSERT CODE HERE;   % Voltage across the load, V
IL = Vth./(Rth+RL);         % Current through the load resistor, A
PL = VL.*IL;                % Power absorbed by the resistor, W

% Plot
clf;
figure(1);
% INSERT CODE HERE
% note that the color and line scheme do not need to match the
 solution.
```

# Solution

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